Sum and Difference Identities

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- $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$
- $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

- $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

- $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
- $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Let's look at specific angles...

$\cos 15°$

$\cos(45-30)$

Change angle to be sum/diff of 2 angles on unit circle

$\cos 45\cos 30+\sin 45\sin 30$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

Rewrite using identity

find the sin/cos of the angles

$\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$

Simplify

$\frac{\sqrt{6}+\sqrt{2}}{4}$

$$\cos 80° \cos 20° + \sin 80° \sin 20°$$

$\cos(80 - 20)$

$\cos 60$

$\frac{1}{2}$

$$\tan\left(\frac{4\pi}{3}-\frac{\pi}{4}\right)$$

Verify using Sum & Difference Identities

$$\sin(\frac{\pi}{2} + x) = \cos x$$

α β

$\sin\frac{\pi}{2}\cos x + \sin x \cos\frac{\pi}{2}$

$1 \cdot \cos x + \sin x \cdot 0$

$\cos x + 0$

$\boxed{\cos x}$

Verify Using Sum and Difference Identities

$$\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$$

$$\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$$

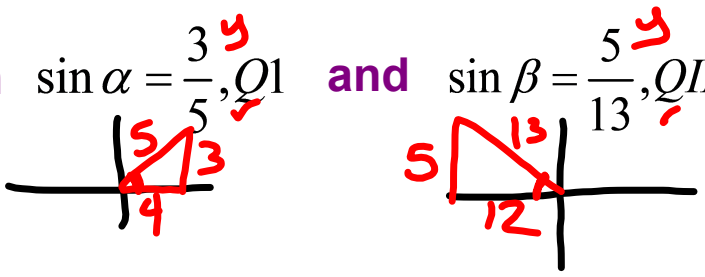
$$\frac{1-\tan\theta}{1+\tan\theta}$$

Verify Using S&D Identities

$$\sin(x+y)+\sin(x-y)=2\sin x\cos y$$

Sum & Difference

Given $\sin\alpha = \frac{3}{5}, QI$ **and** $\sin\beta = \frac{5}{13}, QII$



Find:

1. $\sin(\alpha+\beta)$

$\sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\left(\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = \frac{56}{65}$

2. $\cos(\alpha+\beta)$
3. $\tan(\alpha+\beta)$

Given $\tan\alpha = \frac{-3}{4}$, QII **and** $\cos\beta = \frac{1}{3}$

Find:

1. $\sin(\alpha+\beta)$
2. $\cos(\alpha+\beta)$
3. $\tan(\alpha+\beta)$

hw 5.2 p604

16-24e, 26, 28, 38-44e, 58,60